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R.R.D.E. RESEARCH REPORT NO. 312

1. NOTES ON THE EFFECT OF PHASING ERROR ON THE CONTROL OF A GUIDED PROJECTILE

3. By: C. Mack
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2. 25p.

SUMMARY

A previous report (GP/WORK/5/A28) deals with the cause and magnitude of Phasing Error.

The present report considers the effect of phasing error on the control of a G.P. and is concerned with the design of the control function; Cartesian and Polar Control Systems are considered. It is shown that under appropriate conditions a simple Cartesian Control system can be made stable whilst a simple form of Polar Control proves to be unstable and the more complicated form requires to be studied on a machine.

CONTENTS

1. INTRODUCTION

1.1. Effect of Phasing Error

If a phasing error α is introduced, any control accelerations applied to the rocket are inclined at an angle α with the required direction. Whenever α is greater than 90° the control system will be unstable since instead of trying to restore the G.P. to the radar beam axis it will direct it away.

The effect of phasing error may be reduced either by reducing the phasing error itself (as is considered in GP/WORK/5/A28) or by designing the control function so that it will tolerate a large error.

Full equations of motion relating to the radar control, servo system and aerodynamic system together are not treated here. If displacements are more than some limiting size, non linearity necessarily appears in certain controls making it simpler to investigate the behaviour by means of a simulator (see GP/WORK/5/A47). The following treatment applies only to the linear case and therefore represents performance for small displacements only.

1.2. **Basic Principles**

Throughout the report, consider the radar beam to be stationary. Suppose also that the slant range of the G.P. is sufficiently great for variations of speed parallel to the radar beam axis to be negligible. Hence all that need be considered is the behaviour of the G.P. in a plane at right angles to the beam axis moving with it up the beam.

Consider Fig. 1 which represents this plane, the instantaneous cross-section through the projectile perpendicular to the beam. In the Polar Control system, the G.P. is fitted with ailerons and elevators so that it may bank and climb. In this case the aim is to bank the projectile so that its wings are normal to its displacement from the beam axis, and then cause it to climb or dive towards the axis. Suppose the polar co-ordinates of the projectile are (r,θ) relative to the beam axis as origin, and some fixed initial reference line. Then the radar system has to provide an information θ to enable the projectile to bank and an information r to give the steepness of the climb. This system has a drawback in that the projectile posses a certain roll inertia and cannot respond to a particular information θ by banking instantly.

Fig. 2 represents the situation for the Cartesian Control System. The projectile is maintained from rolling so that the controls act as rudders and elevators enabling it to turn and climb. If the cartesian co-ordinates of the projectile are (x,y) where x is horizontal and y vertical, then the radar system is expected to provide information about x to the rudders and y to the elevators.

The two systems are fundamentally different because of the roll inertia of the Polar Systems. With cartesian control it is possible to move the projectile in any direction instantly - provided that the servos act instantly. However, with polar control the projectile can immediately move only normal to the plane of its wings. Due to roll inertia, an appreciable time will elapse before it is banked in the required direction. To mimic a stable Cartesian system in the Polar Control system would require a complicated control function, and powerful servos to overcome the long time lag near the beam axis.

2. CARTESIAN CONTROL FUNCTIONS

2.1. System with Zero Phasing Error.

Paragraph 1.2. describes the basic principles of this system. To achieve simplicity, suppose that the acceleration $\ddot{y}$ is a function $g(y)$ which does not depend on x. Assume that a similar function is applied horizontally in the x direction so that the acceleration $\ddot{x} = g(x)$.

A simple control is $g(x) = -Ax - B\dot{x}$ 2.1.1.

where A and B are constants, positive for stability.

The equations of motion are:

$$\ddot{x} + B\dot{x} + Ax = 0 \qquad 2.1.2.$$

$$\ddot{y} + B\dot{y} + Ay = 0 \qquad 2.1.3.$$

x is provided by the radar system and $\dot{x}$ is generated from it. A function of these feeds the servo mechanism and rudders which are assumed to act instantly and cause an acceleration $g(x)$.

Rewrite equation 2.1.2. by substituting $T = \beta t$ and denoting $\frac{d}{dT}$ by p.

$$[\beta^2 p^2 + \beta Bp + A] x = 0$$

Since A and B are positive, put $\beta = \sqrt{A}$ and $B = \lambda\beta$

where λ is positive. Then

$$(p^2 + \lambda p + 1)x = 0 \text{ and } (p^2 + \lambda p + 1)\, y = 0 \qquad 2.1.4.$$

The solutions are $x = A_1 \exp(a_1 t) + A_2 \exp(a_2 t)$

$$y = B_1 \exp(a_1 t) + B_2 \exp(a_2 t) \qquad 2.1.5.$$

where $A_1\ A_2\ B_1\ B_2$ are determined by the initial conditions and $a_1\ a_2$ are the roots of $p^2 + \lambda p + 1 = 0$. Both x and y decay to zero in time, because λ is positive and real.

2.2. System with Phasing Error α

When a phase error of α exists, the radar system provides an angle $\theta - \alpha$ instead of θ. Instead of $y = r \sin\theta$, it feeds to the servo $r \sin(\theta - \alpha) = y\cos\alpha - x\sin\alpha$: and instead of $x = r\cos\theta$, it feeds $r\cos(\theta - \alpha) = x\cos\alpha + y\sin\alpha$. See Fig. 3; the effect is the same as if the projectile is banked at an angle α instead of remaining normal to the radius vector.

Hence equation 2.1.2 becomes

$$\ddot{x} + B\dot{x}\cos\alpha + B\dot{y}\sin\alpha + A x\cos\alpha + A y\sin\alpha = 0 \qquad 2.2.1.$$

and so equations 2.1.4. become:-

$$\left[p^2 + \cos\alpha\,(\lambda p + 1)\right] x + \sin\alpha \left[\lambda p + 1\right] y = 0 \qquad 2.2.2.$$

$$\left[p^2 + \cos\alpha\,(\lambda p + 1)\right] y - \sin\alpha \left[\lambda p + 1\right] x = 0 \qquad 2.2.3.$$

Hence $\left[p^4 + 2\lambda\cos\alpha\, p^3 + (2\cos\alpha + \lambda^2)\, p^2 + 2\lambda p + 1\right] x = 0$ 2.2.4.

The solution to this is the sum of terms $R \exp(\gamma t)$ where R is constant and γ is a root of the equation

$$p^4 + 2\lambda\cos\alpha\, p^3 + (2\cos\alpha + \lambda^2)p^2 + 2\lambda p + 1 = 0 \qquad 2.2.5$$

If the real part of γ is positive, this term will increase indefinitely with time and so the motion cannot be stable. Hence all the roots of 2.2.5, must have negative real parts.

Routh's stability criteria give the following inequalities.

$$2\lambda \cos\alpha > 0 \qquad 2.2.6$$

$$2\lambda\,(2\cos^2\alpha + \lambda^2\cos\alpha - 1) > 0 \qquad 2.2.7.$$

$$4\lambda^2\,(\lambda^2\cos\alpha - \sin^2\alpha) > 0 \qquad 2.2.8.$$

Since λ is positive and $\alpha < 90^\circ$, 2.2.6 is satisfied and 2.2.8 includes 2.2.7.

2.3. Stability

For a given λ, stable motion is possible provided that $\alpha < \alpha^1$ where $\lambda^2 \cos\alpha^1 = \sin^2\alpha^1$. α^1 may be called the maximum phasing error for a given λ. Since $\lambda = B/\sqrt{A}$ it is a measure of the amount of derivative control used in the control function.

Amount of derivative control λ	.5	1.0	1.5	2.0	3.0	∞
Maximum phasing error α^1 degrees	28	51.7	67.8	76.3	83.7	90

When $\alpha = 0$; $\lambda = 2$ gives critical damping since the roots of 2.2.5 are then equal. If λ is taken as 2, the control is not sluggish for small phasing errors, and remains stable for phasing errors as large as 75°. Thus it seems desirable that $B = 2\sqrt{A}$

2.4. Settling Down Time

The roots γ of equation 2.2.5, also determine the settling down time. The imaginary part of γ gives the period of the oscillation, while the real part which is negative for stable flight shows the time constant of the settling down.

The complete solution of 2.2.4 is:-

$$x = R_1 \exp(\gamma_1 t) + R_2 \exp(\gamma_2 t) + R_3 \exp(\gamma_3 t) + R_4 \exp(\gamma_4 t) \quad 2.3.1.$$

The coefficients R are determined by initial conditions and so must be taken as arbitrary constants. The settling down time is determined in general by the γ with smallest real part, which corresponds to the longest time constant.

cos α	α	Roots γ of 3.2.5.			
		$\lambda = 2$		Unstable for $\alpha > 51.7^o$	
1.0	0^o	-0.5	$\pm$ i 0.866	-0.5	$\pm$ i 0.866
.9	25.8^o	-0.320	$\pm$ i 1.078	-0.580	$\pm$ i 0.671
.8	36.8^o	-0.195	$\pm$ i 1.180	-0.605	$\pm$ i 0.587
.7	45.6^o	-0.087	$\pm$ i 1.238	-0.613	$\pm$ i 0.523
.65	49.5^o	-0.033	$\pm$ i 1.260	-0.617	$\pm$ i 0.498
.6184	51.7^o	0	$\pm$ i 1.270	-0.618	$\pm$ i 0.486
		$\lambda = 2$		Unstable for $\alpha > 76.3^o$	
1.0	0^o	-1	$\pm$ 0	-1	$\pm$ 0
.9	25.8^o	-1.204	$\pm$ i 1.034	-0.596	$\pm$ i 0.208
.8	36.8^o	-1.032	$\pm$ i 1.360	-0.568	$\pm$ i 0.152
.7	45.6^o	-0.865	$\pm$ i 1.575	-0.535	$\pm$ i 0.145
.6	53.1^o	-0.685	$\pm$ i 1.74	-0.515	$\pm$ i 0.142
.5	60	-0.5	$\pm$ i 1.87	-0.5	$\pm$ i 0.138
.4	66.4^o	-0.312	$\pm$ i 1.98	-0.488	$\pm$ i 0.130
.3	72.5^o	-0.122	$\pm$ i 2.03	-0.478	$\pm$ i 0.071
.237	76.3^o	0	$\pm$ i 2.04	-0.474	$\pm$ i 0.060

Some special cases have been evaluated in Figs 4, 5, 6, 7 with the initial conditions $x = 1$, $\dot{x} = 0$, $y = 0$, $\dot{y} = 0$ and $\lambda = 2$ for the various values of cos α = 0.6, 0.45, 0.3 and 0.25. In the graph x is plotted against T the generalized time, ref para 2.1. For α as large as 65^o, tends to zero very rapidly.

3. POLAR CONTROL FUNCTIONS

3.1. General Equations

In Fig. 1 suppose that the acceleration in climb applied to the G.P. is f(r) a function of r only. It is intended naturally that this should be applied back along the radius vector to restore the projectile to the mirror axis. However the projectile is banked so that in fact it moves at an angle ϕ, in other words at an angle $(\theta - \phi)$ to the radius vector.

The equations of motion are:-

$$\ddot{r} - r\dot{\theta}^2 = -f(r)\cos(\theta - \phi) \quad 3.1.1.$$

$$\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = f(r)\sin(\theta - \phi) \quad 3.1.2.$$

In these two equations there are the three variables r, θ and ϕ. A third equation expresses the form of control applied to the ailerons. The roll inertia cannot be neglected legitimately and appears in the constants of the equation, which represents damped oscillations about an equilibrium position at $\phi = \theta$ and $\dot{\phi} = 0$. Otherwise the function is arbitrary, as any form of damping and restoring may be used.

$$\ddot{\phi} = h(\theta, \phi, \dot{\theta}, \dot{\phi}) \qquad 3.1.3.$$

It is necessary to introduce the variable ϕ in order that the attitude of the projectile may be referred to a fixed direction. Its presence implies that some means of generating it must be provided in the projectile.

3.2. Phasing Error

The effect of a phasing error is that the radar system feeds an angle $\theta - \alpha$ to the controls instead of θ. Equation 3.1.3. shows that now the equilibrium position is at $\phi = \theta - \alpha$, $\dot{\theta} = 0$.

To take an over-simplified case, suppose that the servos are so powerful and roll inertia so small that the system departs from roll equilibrium for only a negligible time. Suppose also that no form of damping is used except relative to $(\dot{\theta} - \dot{\phi})$.

Then $\theta - \phi = \alpha$ does not vary with $\dot{\theta}$. In this case $f(r)\sin(\theta - \phi)$ will be mainly of one sign and so equation 3.1.2. shows that angular momentum about the beam axis will increase indefintely with time. This shows that this simple form of polar control is unstable.

Although the above is only a rough indication, it has been proved rigorously that the system is unstable. Furthermore, the general improvement which might be expected when the effect of gravity and tracking accelerations are considered, is not enough to guarantee stability. Even when the phasing error is zero, the system is not stable, and any time-lags in the servos do not improve the control.

In a general case the three equations 3.1.1, 3.1.2, 3.1.3, cannot be solved formally. Probably a fairly simple polar control function could be made stable, but this is difficult to prove mathematically. If ϕ were made dependent on $\dot{\theta}$, the projectile could be caused to bank in such a way as would tend to make $\dot{\theta} = 0$. This appears to provide a stable system: certainly the angular momentum would remain finite.

4. CONCLUSIONS

It is concluded that a simple Cartesian Control System can be made stable, and that if the control function has longer time constants than the aerodynamic equations of motion, a sufficiently large measure of derivative control will maintain the stability for a phasing error which approaches 90°. If the phasing error is always less than 70°, a moderate amount of derivative control suffices and the control is not sluggish.

A simple form of Polar Control proves to be unstable, and the treatment of more complicated forms is found to be too difficult for a paper analysis. These are best studied on a machine as in fact is being done at Walton.

R.R.D.E.
Malvern, Worcs.
5th July, 1946.

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FIG. 1, 2 & 3.

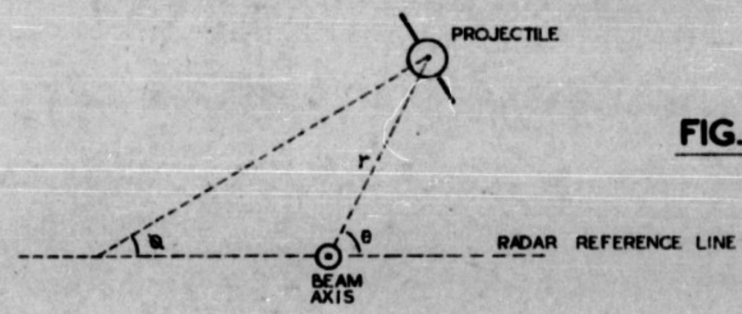

FIG. 1.

FIG. 2.

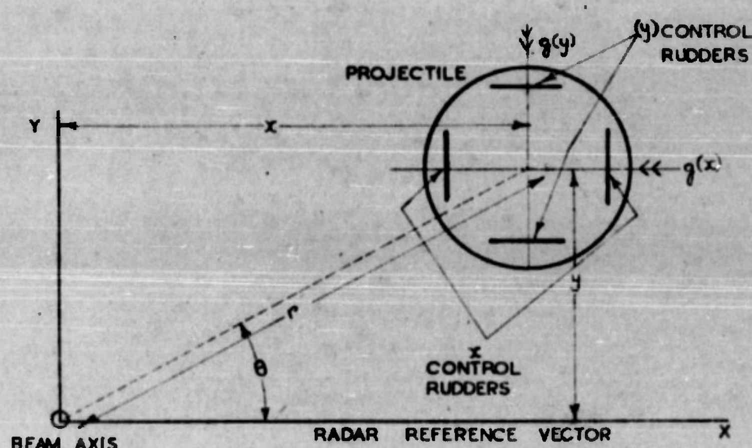

FIG. 3.

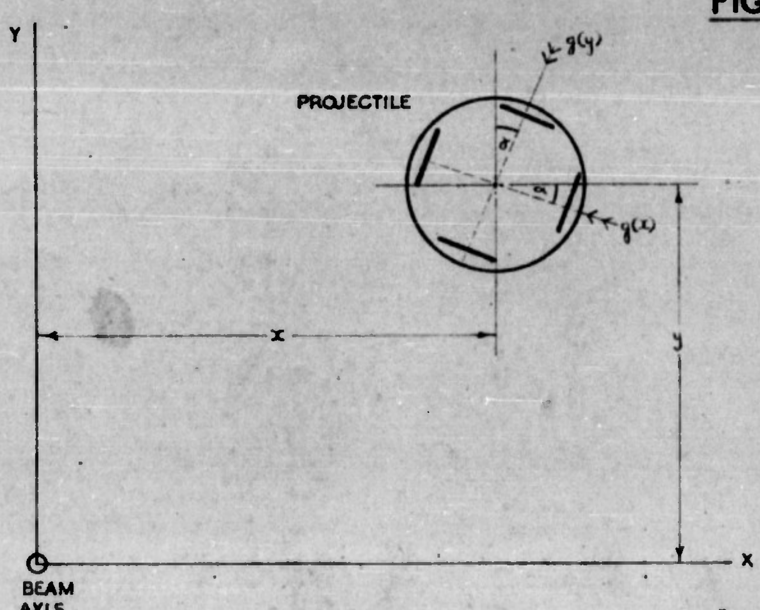

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FIG.4.

SOLUTION OF
$$\begin{cases} \ddot{x} + (B\dot{x} + Ax)\cos\alpha + (B\dot{y} + Ay)\sin\alpha = 0 \\ \ddot{y} + (B\dot{y} + Ay)\cos\alpha - (B\dot{x} + Ax)\sin\alpha = 0 \end{cases}$$

WITH INITIAL CONDITIONS $x = 1,\ \dot{x} = y = \dot{y} = 0$

$B/\sqrt{A} = \lambda = 2$

WITH $\cos\alpha = \cdot 6$, $\alpha = 53\cdot 1^{\circ}$

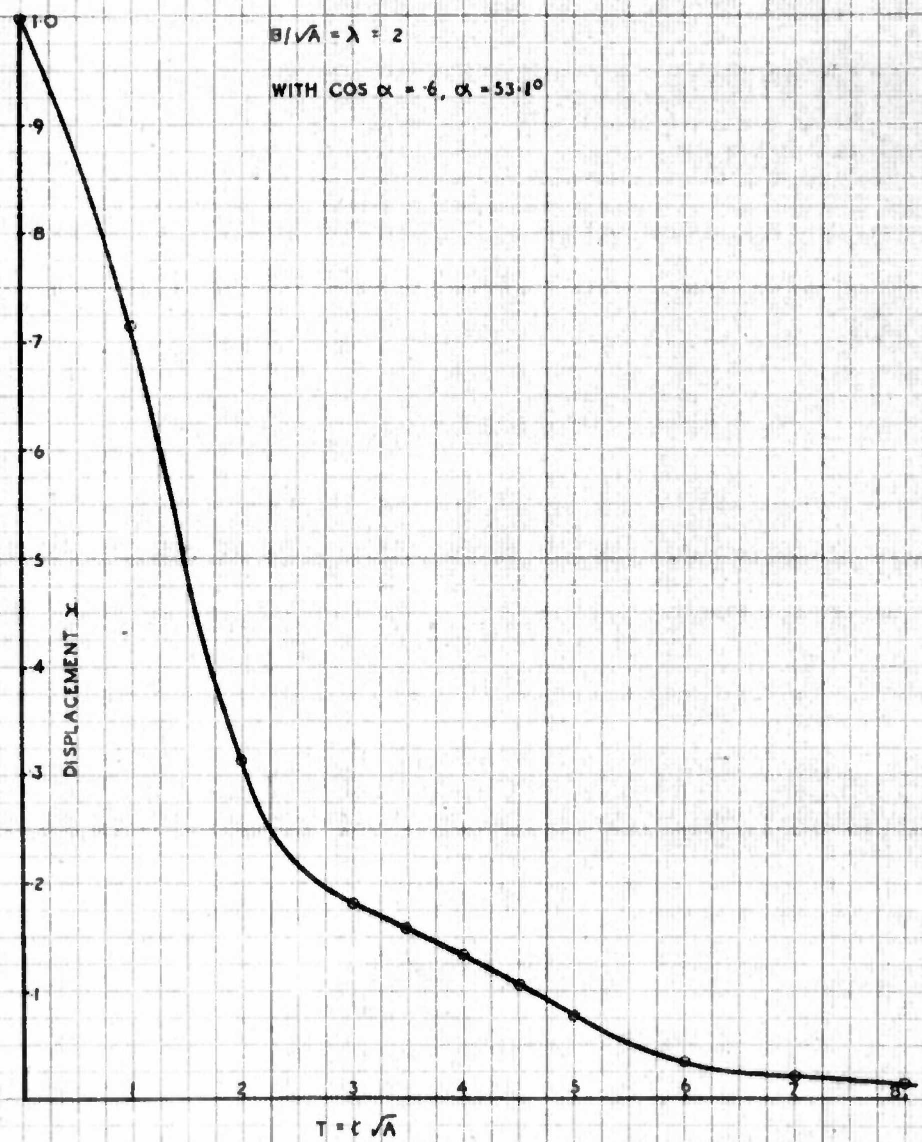

FIG. 5.

SOLUTION OF
$$\left\{\begin{array}{l}\ddot{x} + (B\dot{x} + Ax)\cos\alpha + (B\dot{y} + Ay)\sin\alpha = 0 \\ \ddot{y} + (B\dot{y} + Ay)\cos\alpha - (B\dot{x} + Ax)\sin\alpha = 0\end{array}\right\}$$

WITH INITIAL CONDITIONS $x = 1, \dot{x} = y = \dot{y} = 0$

$B\sqrt{A} = \lambda = 2$

WITH $\cos\alpha = \cdot 45$, $\alpha = 63\cdot 2^\circ$

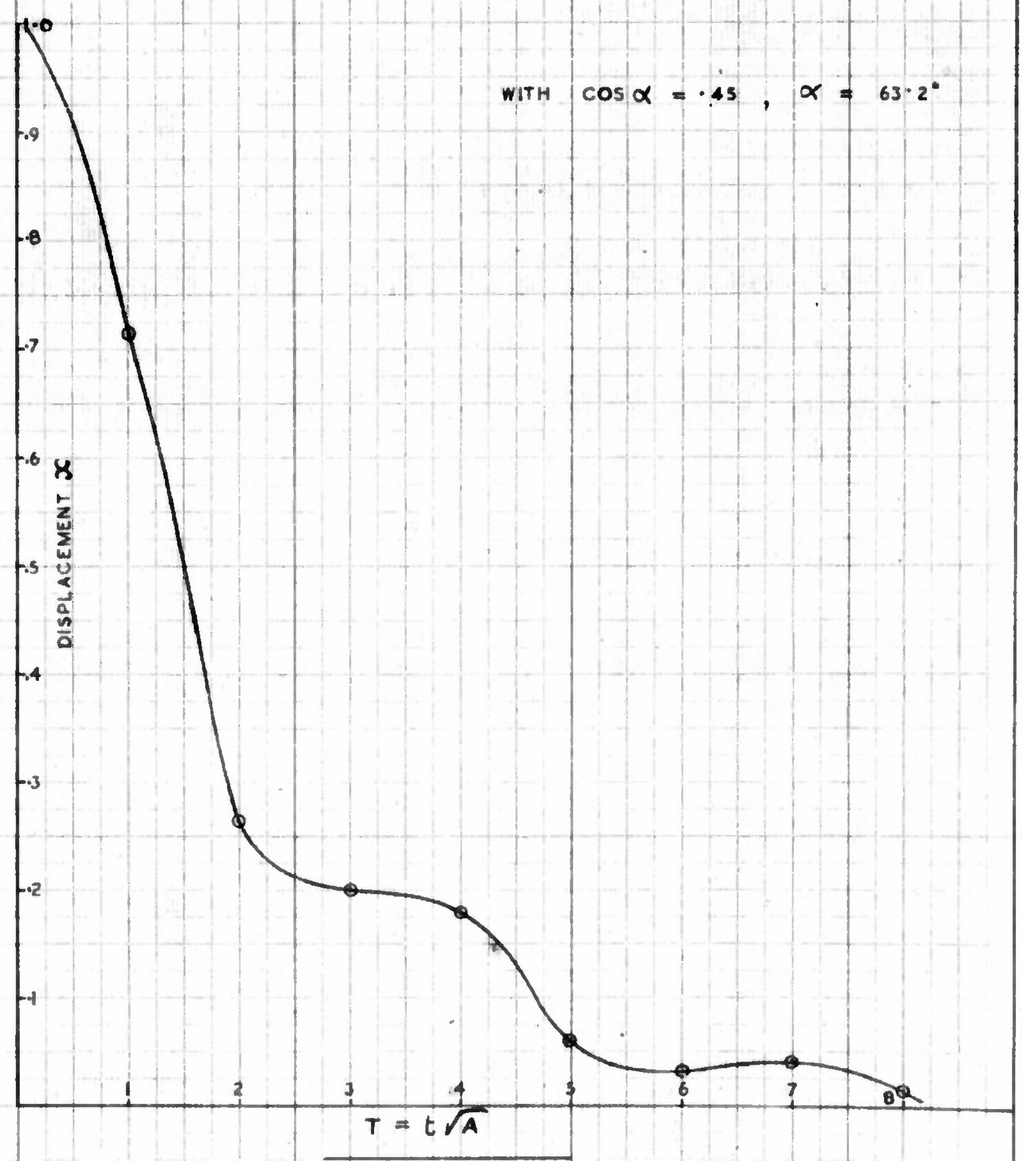

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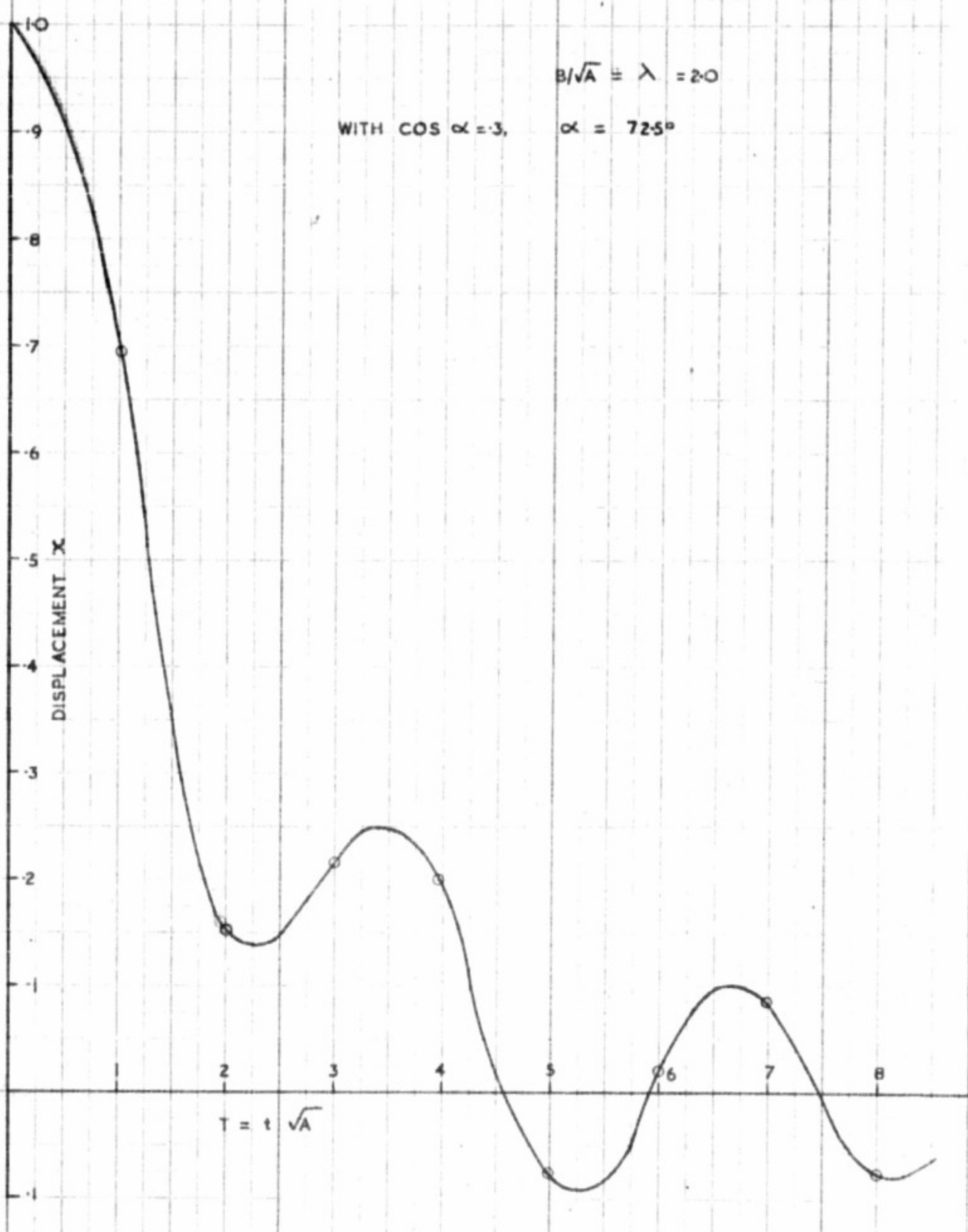
FIG. 6.

SOLUTION OF
$$\begin{cases} \ddot{x} + (B\dot{x} + Ax)\cos\alpha + (B\dot{y} + Ay)\sin\alpha = 0 \\ \ddot{y} + (B\dot{y} + Ay)\cos\alpha - (B\dot{x} + Ax)\sin\alpha = 0 \end{cases}$$

WITH INITIAL CONDITIONS $x=1$, $\dot{x} = y = \dot{y} = 0$.



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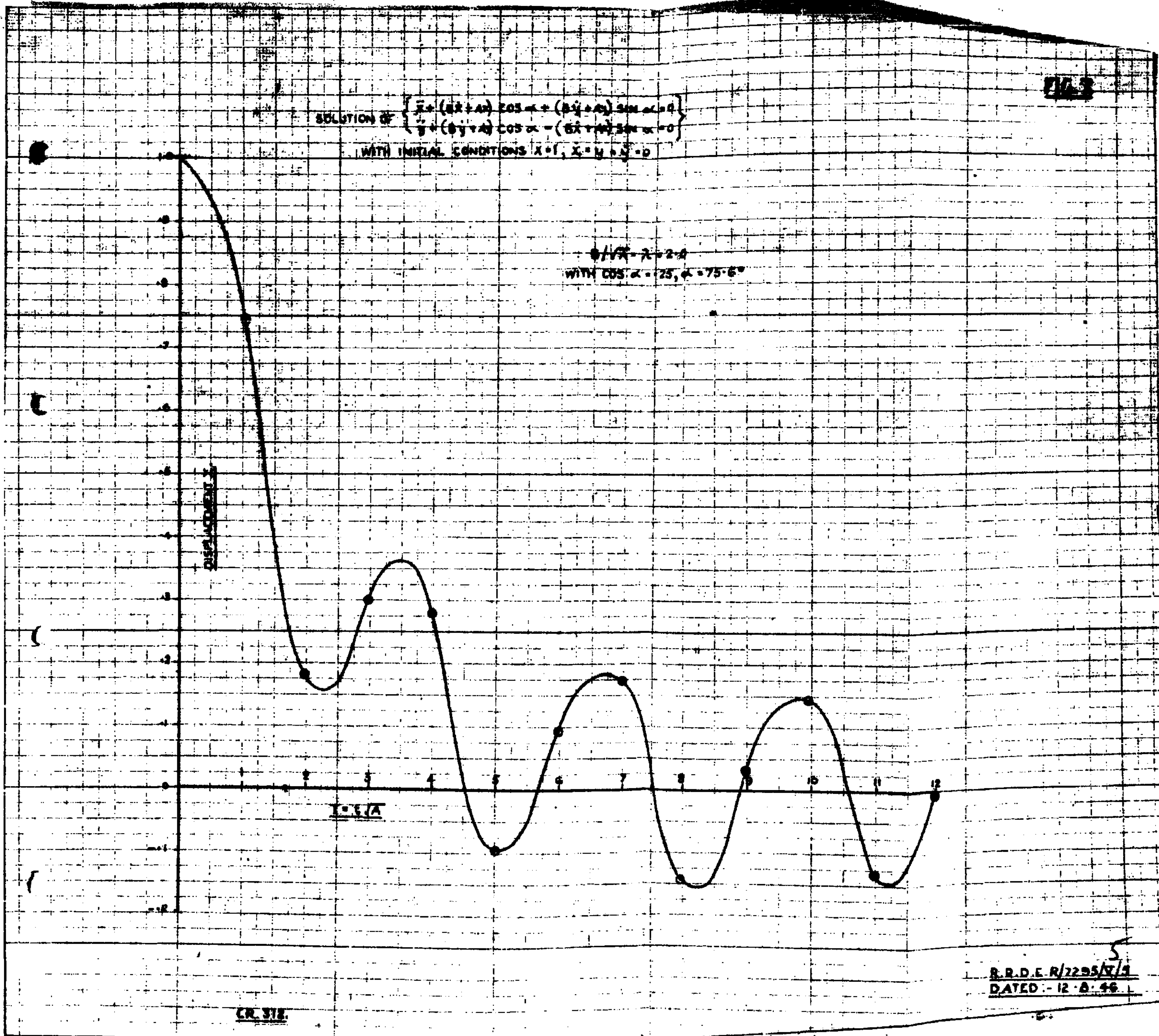
FIG. 3
SOLUTION OF { ẍ + (Bẋ + Ax) cos α + (Bẏ + Ay) sin α = 0 ; ÿ + (Bẏ + Ay) cos α − (Bẋ + Ax) sin α = 0 }
WITH INITIAL CONDITIONS x = 1, ẋ = y = ẏ = 0
B/√A = λ = 2·0
WITH COS α = ·25, α = 75·6°
DISPLACEMENT x
T = t√A
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ABSTRACT:

The effects of phasing errors may be reduced either by reducing the phasing error itself, or by designing controls that will function with great tolerance. The design of Cartesian and Polar Control Systems for control of aerial missiles is described. With Cartesian Control, it is possible to control the projectile in any direction instantly, provided the servos act instantly. However, with Polar Control the projectile can move immediately only normal to the plane of its wing. It is shown in the research report that a simple Cartesian Control System proves stable while the Polar Control System proves unstable.

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